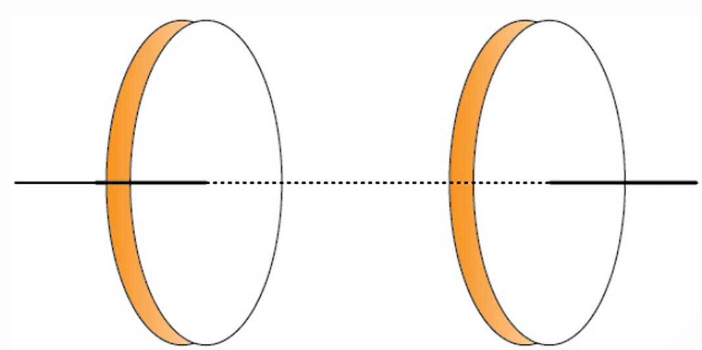
Q 8.1) The Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.

(a) Calculate the capacitance and the rate of change of the potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



Answer 8.1:

Given Values:

The radius of each circular plate (r) is 12 cm or 0.12 m

The distance between the plates (d) is 5 cm or 0.05 m

The charging current (I) is 0.15 A

The permittivity of free space is $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

(a) The capacitance between the two plates can be calculated as follows:

$$C = \frac{\varepsilon_0 A}{d}$$

where,

A = Area of each plate = πr^2 $C = \frac{\varepsilon_0 \pi r^2}{d}$

$$= \frac{8.85 \times 10^{-12} \times \pi (0.12)^2}{0.05}$$

$$= 8.0032 \times 10^{-12}\ F$$

= 80.032 pF

The charge on each plate is given by,

q = CV

where,

V is the potential difference across the plates

Differentiation on both sides with respect to time (t) gives:

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

But, $\frac{dq}{dt}$ = Current (I)

$\therefore \ \frac{dV}{dt} = \frac{I}{C}$

=> $\frac{0.15}{80.032 \times 10^{-12}} = 1.87 \times \ 10^{9} \ V/s$

Therefore, the change in the potential difference between the plates is $1.87 \times \ 10^{9} \ V/s$.

(b) The displacement current across the plates is the same as the conduction current. Hence, the displacement current, id is 0.15 A.

(c) Yes

Kirchhoff's first rule is valid at each plate of the capacitor provided that we take the sum of conduction and displacement for current.

Q 8.2) A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with an (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

Answer 8.2:

Radius of each circular plate, R = 6.0 cm = 0.06 m

Capacitance of a parallel plate capacitor, C = 100 pF = $100 \times \ 10^{-12} \ F$

Supply voltage, V = 230 V

Angular frequency, $\omega = \ 300 \ rad \ s^{-1}$

(a) Rms value of conduction current, I = $\frac{V}{X_c}$

Where,

X_c = Capacitive reactance

= $\frac{1}{\omega C} \ \therefore \ I = \ V \times \ \omega C$

= $230 \times \ 300 \times \ 100 \times \ 10^{-12}$

= $6.9 \times \ 10^{-6} A$

= $6.9 \ \mu A$

Hence, the rms value of conduction current is $6.9 \ \mu A$.

(b) Yes, conduction current is equivalent to displacement current.

(c) Magnetic field is given as:

$B = \ \frac{\mu_0 r}{2\pi R^2} I_0$

Where,

μ_0 = Permeability of free space = $4\pi \ \times \ 10^{-7} \ N \ A^{-2}$ I_0 = Maximum value of current = $\sqrt{2} \ I$

r = Distance between the plates from the axis = 3.0 cm = 0.03 m

$\therefore\ B = \frac{4\pi\times10^{-7}\times0.03\times\sqrt{2}\times6.9\times10^{-6}}{2\pi\times(0.06)^2}$

= $1.63\times\ 10^{-11}\ T$

Hence, the magnetic field at that point is $1.63\times\ 10^{-11}\ T$.

Q 8.3) What physical quantity is the same for X-rays of wavelength 10^{-10}m, the red light of wavelength 6800 Å and radiowaves of wavelength 500m?

Answer 8.3:

The speed of light ($3\times\ 10^8$ m/s) in a vacuum is the same for all wavelengths. It is independent of the wavelength in the vacuum.

Q 8.4) A plane electromagnetic wave travels in vacuum along the z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Answer 8.4:

The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (H) are in the x-y plane. They are mutually perpendicular. Frequency of the wave, v = 30 MHz = $30\times\ 10^6\ s^{-1}$

Speed of light in vacuum, C = $3\times\ 10^8$ m/s

Wavelength of a wave is given a:

$\lambda = \frac{c}{v}$

= $\frac{3\times10^8}{30\times10^6}$ = 10 m

Q 8.5) A radio can tune in to any station in the 7.5 MHz to 12 MHz bands. What is the corresponding wavelength band?

Answer 8.5:

A radio can tune to minimum frequency, $v_1 = 7.5\ MHz = 7.5\times\ 10^6\ Hz$

Maximum frequency, $v_2 = 12\ MHz = 12\times\ 10^6\ Hz$

Speed of light, c = $3\times\ 10^8\ m/s$

Corresponding wavelength for v_1 can be calculated as:

$\lambda_1 = \frac{c}{v_1}\ \frac{3\times10^3}{7.5\times10^6} = 40\ m$

Corresponding wavelength for v_2 can be calculated as:

$\lambda_2 = \frac{c}{v_2}\ \frac{3\times10^3}{12\times10^6} = 25\ m$

Thus, the wavelength band of the radio is 40 m to 25 m.

Q 8.6) A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Answer 8.6:

The frequency of an electromagnetic wave produced by the oscillator is the same as that of a charged particle oscillating about its mean position i.e., 10^9 Hz.

Q 8.7) The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0=510 nT. What is the amplitude of the electric field part of the wave?

Answer 8.7:

Amplitude of magnetic field of an electromagnetic wave in a vacuum,

$$B_0 = 510\ nT = 510 \times 10^{-9}\ T$$

Speed of light in vacuum, c = $3 \times 10^8\ m/s$

Amplitude of electric field of an electromagnetic wave is given by the relation,

$$E = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153\ N/C$$

Therefore, the electric field part of the wave is 153 N/C.

Q 8.8) Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120\ N/C$ and that its frequency is v = 50 MHz.(a)

Determine $B_0,\ \omega,\ k\ and\ \lambda$ (b) Find expressions for E and B.

Answer 8.8:

Electric field amplitude, $E_0 = 120\ N/C$

Frequency of source, v = 50 MHz = 50×10^6 Hz

Speed of light, c = 3×10^8 m/s

(a) Magnitude of magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

= $\frac{120}{3 \times 10^8}$

= $4 \times 10^{-7}\ T = 400\ nT$

Angular frequency of source is given by:

$$\omega = 2nv = 2n \times 50 \times 10^6$$

= 3.14×10^8 rad/s

Propagation constant is given as:

$$k = \frac{\omega}{c}$$

= $\frac{3.14\times10^8}{3\times10^8} = 1.05\ rad/m$

Wavelength of wave is given by:

$$\lambda = \frac{c}{v}$$

= $\frac{3\times10^8}{50\times10^6}$ = 6.0 m

(b) Suppose the wave is propagating in the positive x-direction. Then, the electric field vector will be in the positive y-direction and the magnetic field vector will be in the positive z-direction. This is because all three vectors are mutually perpendicular.

Equation of electric field vector is given as:

$$\overline{E} = E_0\ sin(kx – \omega t)\ \widehat{j}$$

= $120\ sin[1.05x – 3.14 \times\ 10^8 t]\ \widehat{j}$

And, magnetic field vector is given as:

$$\overline{B} = B_0\ sin(kx – \omega t)\ \widehat{k}\ \overline{B} = (4 \times\ 10^{-7})\ sin[1.05x – 3.14 \times\ 10^8 t]\ \widehat{k}$$

Q 8.9) The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hv (for the energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for
different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Answer 8.9:

The energy of a photon is given as:

E = hv = $\frac{hc}{\lambda}$

Where,

h = Planck's constant = $6.6 \times\ 10^{-34}\ Js$

c = Speed of light = $3 \times\ 10^8\ m/s$ λ = Wavelength of radiation

$$\therefore\ E = \frac{6.6\times10^{-34}\times3\times10^8}{\lambda} = \frac{19.8\times10^{-26}}{\lambda}\ J$$

= $\frac{19.8\times10^{-26}}{\lambda\times1.6\times10^{-19}} = \frac{12.375\times10^{-7}}{\lambda}\ eV$

The given table lists the photon energies for different parts of an electromagnet spectrum for different λ.

λ (m)	10^3	1	10^{-3}	10^{-6}	10^{-8}	10^{-10}	10^{-12}
E (eV)	12.375×10^{-10}	12.375×10^{-7}	12.375×10^{-4}	12.375×10^{-1}	$12.375 \times\ 10^1$	$12.375 \times\ 10^3$	$12.375 \times\ 10^5$

The photon energies for the different parts of the spectrum of a source indicate the spacing of the relevant energy levels of the source

Q 8.10) In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz and amplitude 48 V m^{-1}.
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?

(c) Show that the average energy density of the E field equals the average energy density of the B field. [c = $3 \times 10^8\ m\ s^{-1}$]

Answer 8.10:

Frequency of the electromagnetic wave, v = $2 \times 10^{10}\ Hz$

Electric field amplitude, $E_0 = 48\ V\ m^{-1}$

Speed of light, c = $3 \times 10^8\ m/s$

(a) Wavelength of a wave is given as:

$\lambda = \frac{c}{v}$

= $\frac{3\times10^8}{2\times10^{10}} = 0.015\ m$

(b) Magnetic field strength is given as:

$B_0 = \frac{E_0}{c}$

= $\frac{48}{3\times10^8} = 1.6 \times 10^{-7}\ T$

(c) Energy density of the electric field is given as:

$U_E = \frac{1}{2}\ \epsilon_0\ E^2$

And, energy density of the magnetic field is given as:

$U_B = \frac{1}{2\mu_0} B^2$

Where,

ϵ_0 = Permittivity of free space

μ_0 = Permeability of free space

E = cB ...(1)

Where,

$c = \frac{1}{\sqrt{\epsilon_0\ \mu_0}}$ ...(2)

Putting equation (2) in equation (1), we get

$E = \frac{1}{\sqrt{\epsilon_0\ \mu_0}}\ B$

Squaring on both sides, we get $E^2 = \frac{1}{\epsilon_0\ \mu_0}\ B^2\ \ \epsilon_0\ E^2 = \frac{B^2}{\mu_0}\ \ \frac{1}{2}\ \epsilon_0\ E^2 = \frac{1}{2}\ \frac{B^2}{\mu_0}$

=> $U_E = U_B$